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II. Solution by **MARCUS BAKER**, U. S. Geological Survey, Washington, D. C.

Designate the perpendiculars from A, B, C by h', h'', h'''; then from similar triangles,

$$p':a::h'-2r:h', \text{ or } \frac{p'}{a}=1-2\frac{r}{h'}.$$

Similarly, $\frac{p''}{b}=1-2\frac{r}{h''}$ and $\frac{p'''}{c}=1-2\frac{r}{h'''}$; whence

$$\frac{p'}{a}+\frac{p''}{b}+\frac{p'''}{c}=3-2\left(\frac{r}{h'}+\frac{r}{h''}+\frac{r}{h'''}\right)=1\ldots\ldots(1),$$

$$\left(\text{since, } \frac{1}{h'}+\frac{1}{h''}+\frac{1}{h'''}=\frac{1}{r}\right).$$

Similarly, if P', P'', P''', denote the length of the tangent to the *escribed* circles

$$\frac{P'}{a}+\frac{P''}{b}+\frac{P'''}{c}=3+2\left(\frac{r'}{h'}+\frac{r''}{h''}+\frac{r'''}{h'''}\right)\ldots\ldots(2).$$

Again, for a *tetrahedron*, let p', p'', p''', p'''', designate the planes drawn tangent to the inscribed sphere and parallel to the faces a, b, c, d; and P', P'', P'', P''' the plane similarly drawn tangent to the escribed sphere. Then

$$\sqrt{\frac{p'}{a}}+\sqrt{\frac{p''}{b}}+\sqrt{\frac{p'''}{c}}+\sqrt{\frac{p''''}{d}}=4-2\left(\frac{r}{h'}+\frac{r}{h''}+\frac{r}{h'''}+\frac{r}{h''''}\right)=2\ldots\ldots(3).$$

$$\sqrt{\frac{P'}{a}}+\sqrt{\frac{P'}{b}}+\sqrt{\frac{P''}{c}}+\sqrt{\frac{P'''}{d}}=4+2\left(\frac{r'}{h'}+\frac{r''}{h''}+\frac{r'''}{h'''}+\frac{r''''}{h''''}\right)\ldots\ldots(4).$$

Also solved by ***G. B. M. ZERR*** and ***H. C. WHITAKER***.

GEOMETRY.

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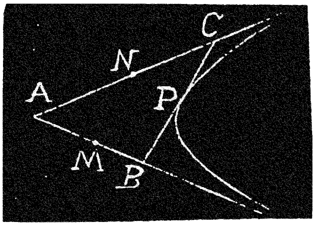
164. Proposed by **J. M. HARCOURT**, M. D., 306 Clinton Street, Brooklyn, N. Y.

Given two tangents to a parabola, find the locus of the center of the nine-point circle of the triangle by the two given tangents and any third tangent.

Solution by **J. SCHEFFER**, A. M., Hagerstown, Md.



Let the equation of the parabola be $y^2=4mx$; then will the equation of three tangents be of the form $y=ax+\frac{m}{a}\ldots\ldots(1)$, $y=a'x+\frac{m}{a'}\ldots\ldots(2)$, $y=a''x+\frac{m}{a''}\ldots\ldots(3)$; the equation (1) being that of AB, (2) of AC, and (3) of BC.

The equation of the altitude from A upon

BC will be

$$aa'(x+a''y)=m(1+a'a''+a''a)......(4),$$

and that of *C* upon *AB*,

$$a'a''(x+ay)=m(1+a''a+aa'')......(5).$$

Combining (4) and (5), we find the co-ordinates of the orthocenter of $\triangle ABC$ to be $x=-m$, $y=m\left(\frac{1}{aa'a''}+\frac{1}{a}+\frac{1}{a'}+\frac{1}{a''}\right)......(6)$.

The symmetry with reference to a, a', a'' shows that the three altitudes all pass through a single point, and the value of x shows that the locus of the orthocenter of *any* variable tangential triangle whatever is the directrix of the parabola. Since the co-ordinates of *A*, *B*, and *C* are respectively,

$$\frac{m}{aa'},\ m\left(\frac{1}{a}+\frac{1}{a'}\right);\ \frac{m}{aa''},\ m\left(\frac{1}{a}+\frac{1}{a''}\right);\ \frac{m}{a'a''},\ m\left(\frac{1}{a'}+\frac{1}{a''}\right)......(7),$$

we find for the middle points *M*, *N*, *P* of the sides *AB*, *AC*, *BC*, respectively, the co-ordinates

$$\frac{m}{2a}\left(\frac{1}{a'}+\frac{1}{a''}\right),\ \frac{m}{2}\left(\frac{2}{a}+\frac{1}{a'}+\frac{1}{a''}\right);\ \frac{m}{2a'}\left(\frac{1}{a}+\frac{1}{a''}\right),\ \frac{m}{2}\left(\frac{2}{a'}+\frac{1}{a}+\frac{1}{a''}\right);$$

$$\frac{m}{2a''}\left(\frac{1}{a}+\frac{1}{a'}\right),\ \frac{m}{2}\left(\frac{2}{a''}+\frac{1}{a}+\frac{1}{a'}\right)......(8).$$

Consequently the equation of the perpendicular erected at *M* and *N*, respectively, is

$$y+\frac{1}{a}x=\frac{m}{2}\left[\frac{2}{a}+\left(1+\frac{1}{a^2}\right)\left(\frac{1}{a'}+\frac{1}{a''}\right)\right]$$

$$\text{and } y+\frac{1}{a'}x=\frac{m}{2}\left[\frac{2}{a'}+\left(1+\frac{1}{a'^2}\right)\left(\frac{1}{a}+\frac{1}{a''}\right)\right]\ .....(9).$$

Therefore, co-ordinates of their point of intersection, or the center of the circumcircle

$$a=\frac{m}{2}\left(1+\frac{1}{aa'}+\frac{1}{aa''}+\frac{1}{a'a''}\right),\ y=\frac{m}{2}\left(\frac{1}{a}+\frac{1}{a'}+\frac{1}{a''}-\frac{1}{aa'a''}\right)......(10).$$

Since the center of the nine-point circle is the middle point of the straight line that connects the center of the circumcircle with the orthocenter, we find for the co-ordinates of the center of the nine-point circle

$$x=\frac{m}{4}\left(-1+\frac{1}{aa'}+\frac{1}{aa''}+\frac{1}{a'a''}\right),\ y=\frac{m}{4}\left[3\left(\frac{1}{a}+\frac{1}{a'}+\frac{1}{a''}\right)+\frac{1}{aa'a''}\right]\text{..}\ (11)$$

Considering AB and CD the given tangents, therefore a and a' given quantities, and a'' variable, we have in (11) only to eliminate a'' to obtain the equation of the required locus. Thus, putting for the sake of brevity $1/a+1/a'=a$, $1/aa'=b$, we find the equation of the locus to be

$$4ay-4(3-b)x=m[3(1+a^2)-b(4-b)].$$

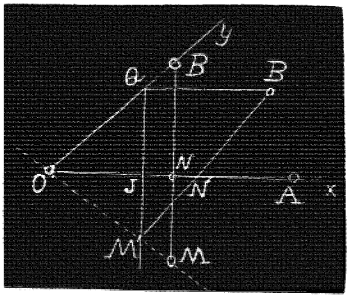
Consequently, the locus is a straight line.

Also solved by *G. B. M. ZERR.*

165. Proposed by **W. H. ECHOLS. B.Sc.. S.E.**, Professor of Mathematics. University of Virginia. Charlottsville, Va.

$OB=b$, $OA=a$ are the semi-conjugate diameters of an ellipse. Draw BM perpendicular to and equal to OA, cutting it in N. Show that as M slides on the fixed line OM and N on OA the point B traces the curve.

Solution by the **PROPOSER.**



Let $M'N'B'$ be an arbitrary position of the line. Draw $B'Q$ parallel to Ox, join $M'Q$. Then

$$\frac{M'J}{JQ}=\frac{M'N'}{N'B'}=\frac{MN}{NB}.$$

$\therefore$ $M'Q$ is perpendicular to Ox. Hence, if $B'Q=x$, $OQ=y$, we have

$$x^2+(M'Q)^2=a^2,\ \text{or}\ x^2+\frac{a^2}{b^2}y^2=a^2.$$

$\therefore$ $x^2/a^2+y^2/b^2=1$. Q. E. D.

166. Proposed by **S. F. NORRIS.** Professor of Astronomy and Mathematics. Baltimore City College, Baltimore, Md.

Two cities are 200 miles apart. To what height must a man ascend from one city in order that he may see the other, supposing the circumference of the earth to be 25,000 miles? [From Wentworth's *New Plane and Solid Geometry*, page 381, No. 619.] Required solution by Geometry.

Solution by **DANIEL NORTHROP.** Mandana, N. Y.

Let A be the position of one city and B the position of the second, distant on a straight line from A, $a=200$ miles. Let P be the position of the man above A when just able to see the city B, h his height above A. Draw the line PA and extend it through the center of the earth to the point D opposite A. Draw the line PB. Then we have $PB^2=PD\times PA$, or $PB^2=(h+2R)h$.